

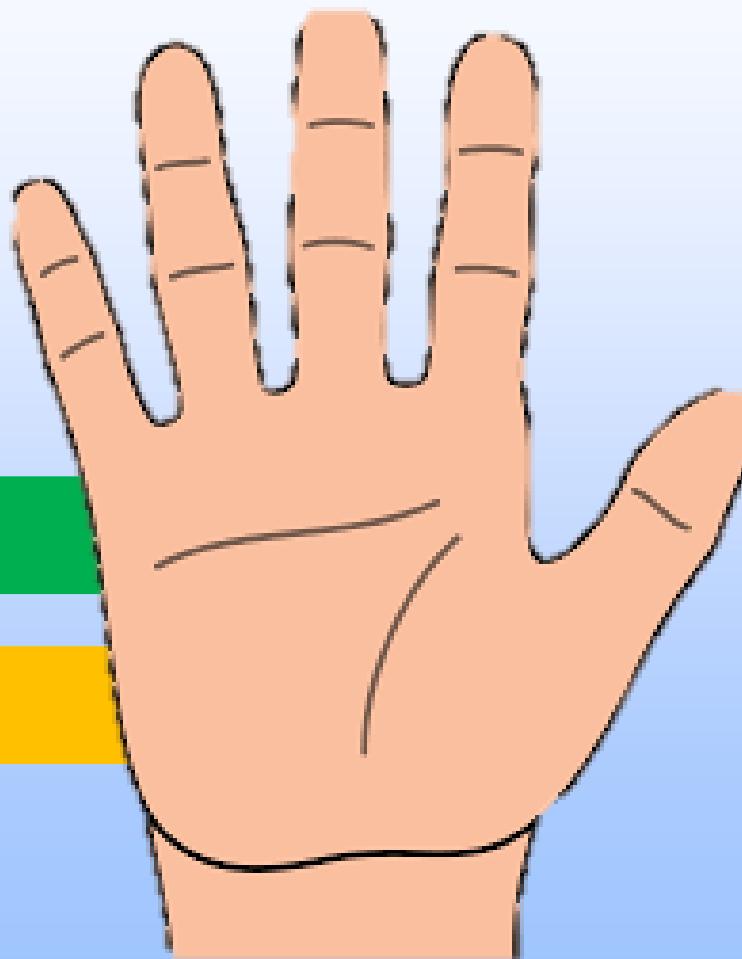


Which line is longer?



$\frac{1}{2}$

$\frac{1}{3}$



Maths: Teaching for Mastery

Ciara Sutton Fitzpatrick and Gemma Field

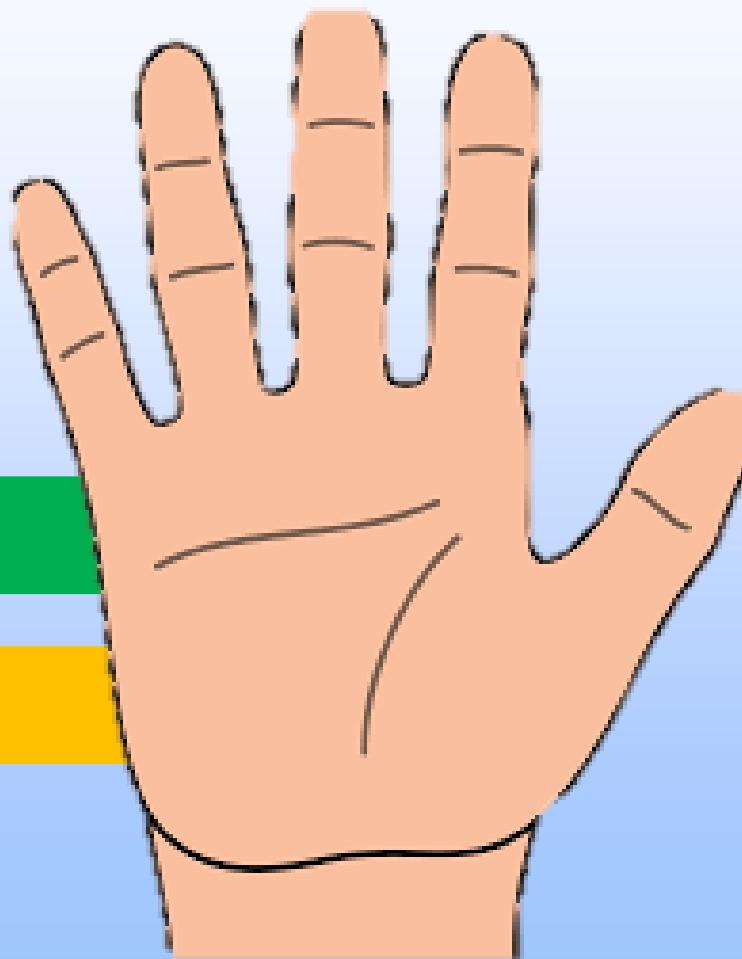


Which line is longer?



$\frac{1}{2}$

$\frac{1}{3}$





What does it mean to master something?

- I know how to do it
- It becomes automatic and I don't need to think about it- for example driving a car
- I'm really good at doing it – painting a room, or a picture
- I can show someone else how to do it.

The Mastery Approach

- ▶ Teaching for mastery rejects the idea that lots of people “can’t do” maths.
- ▶ Background – Singapore and Shanghai maths teaching.
- ▶ Teaching maths in a way that children gain a solid understanding of each concept they are learning before moving onto the next one.
- ▶ Each lesson has one clear objective.
- ▶ Children moving at broadly the same pace.
- ▶ Frequent use of concrete and pictorial resources
- ▶ Procedural fluency and conceptual understanding are emphasised.
- ▶ Key skills are taught for longer and deeper.
- ▶ Immediate interventions for any children falling behind.



This evening we will look at:

- ▶ Mastery Teaching – our aims.
- ▶ Different types of mathematical knowledge
- ▶ Microscopic Progression
- ▶ Concrete – Pictorial – Abstract Approach
- ▶ A typical lesson: Variation, questioning, ‘ping-pong’ teaching and talk for maths.
- ▶ Supporting all learners in a mastery classroom.



Mastery Teaching – Our aims

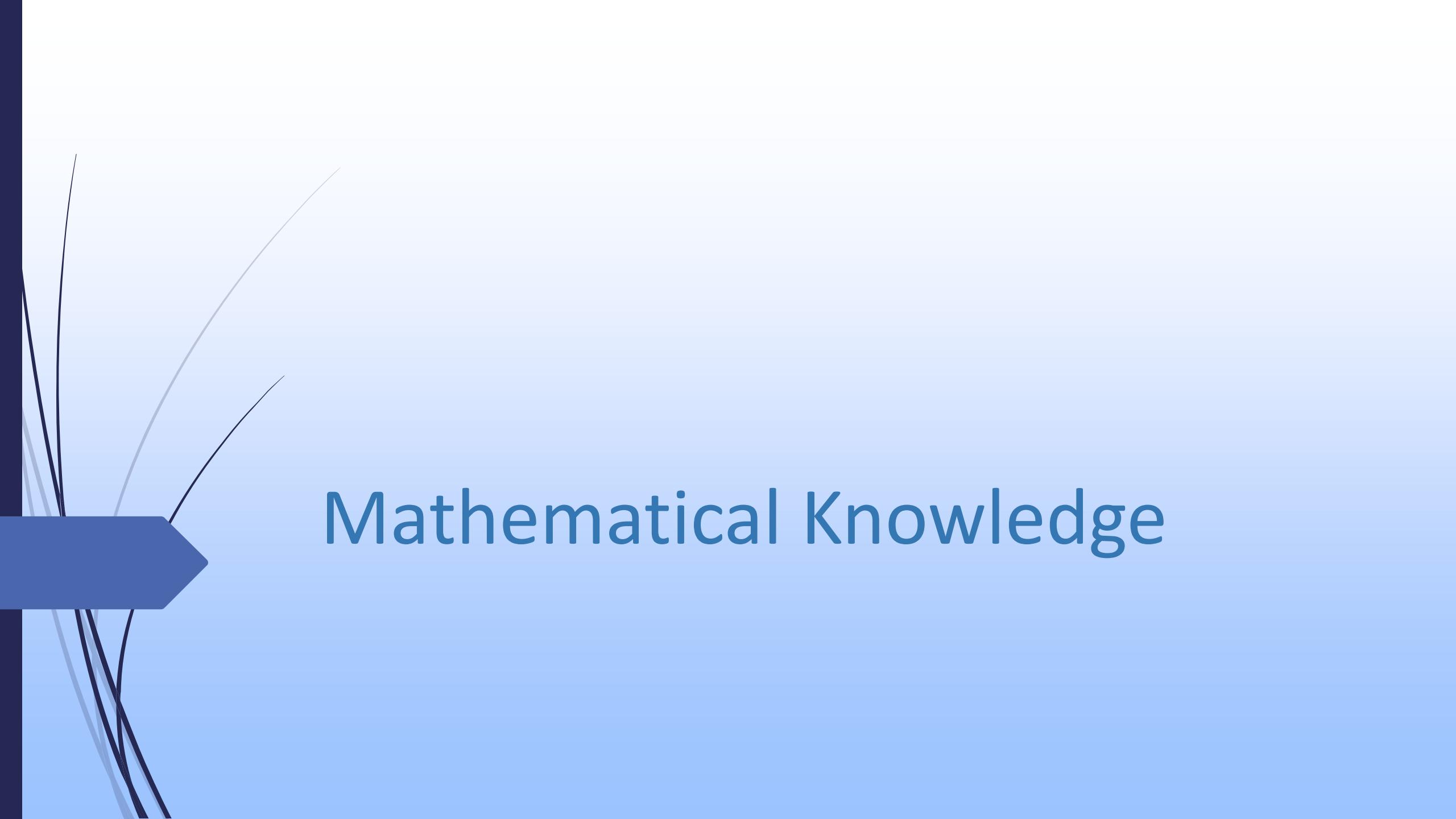
Mastery teaching

- ▶ The background is in Singapore and Shanghai.
- ▶ It has been noticed that children in Singapore and Shanghai keep learning maths for longer and feel more confident than their UK counterparts.
- ▶ We have noticed, not just in our school, gaps appearing from an early age in children's mathematical understanding.
- ▶ Children who were quick graspers were being accelerated quickly through the curriculum without allowing them to secure a deep understanding of each concept.
- ▶ Children who struggled with maths were given easier tasks and did not always access all of the curriculum that the quick graspers did.
- ▶ We aim through teaching for mastery for all children to become confident mathematicians and feel that they can do maths.

Always, Sometimes or Never?



- ➡ To multiply a number by 10, I can add a zero to that number and I'll get the answer



Mathematical Knowledge

Different types of knowledge

Teaching for mastery involves the development of three forms of knowledge:

Factual – I know that

Procedural – I know how

Conceptual – I know why



Factual – I know that

Procedural – I know how

Conceptual – I know why

Factual knowledge:

I know that $\frac{1}{2}$ of 10 is 5

Procedural knowledge:

If I divide 10 by 2 I will get 5

Conceptual knowledge:

I understand to find half of a number you need to divide it into 2 equal parts because a half is something divided into 2 equally. If I divide 10 into 2 equal parts I'll have 5 in each. Look I can prove it!



Factual – I know that
Procedural – I know how
Conceptual – I know why

Factual knowledge:

I know that $0.25 \times 100 = 25$

Procedural knowledge:

I can do this by moving the digits two places to the left.

Conceptual knowledge:

Each place value position gets 10 times bigger as you go to the left. So to make 0.2 100 times bigger I'll have to move the digits two jumps to the left.



Developing procedural knowledge and conceptual understanding

- ▶ Children are frequently asked to prove their answers.
- ▶ We never just tell the children to use a method without explaining why.
- ▶ Children are often led to come up with the method by themselves.
- ▶ Concrete and pictorial resources are used to help support their understanding.
- ▶ Teachers are always asking “Why?”, “How do you know?” and “Are you sure?!”

$$45 + 26 =$$

$$38 + 24 = 62$$

Children explaining
understanding

Ten
ix
you had to go and
would earnall 46.

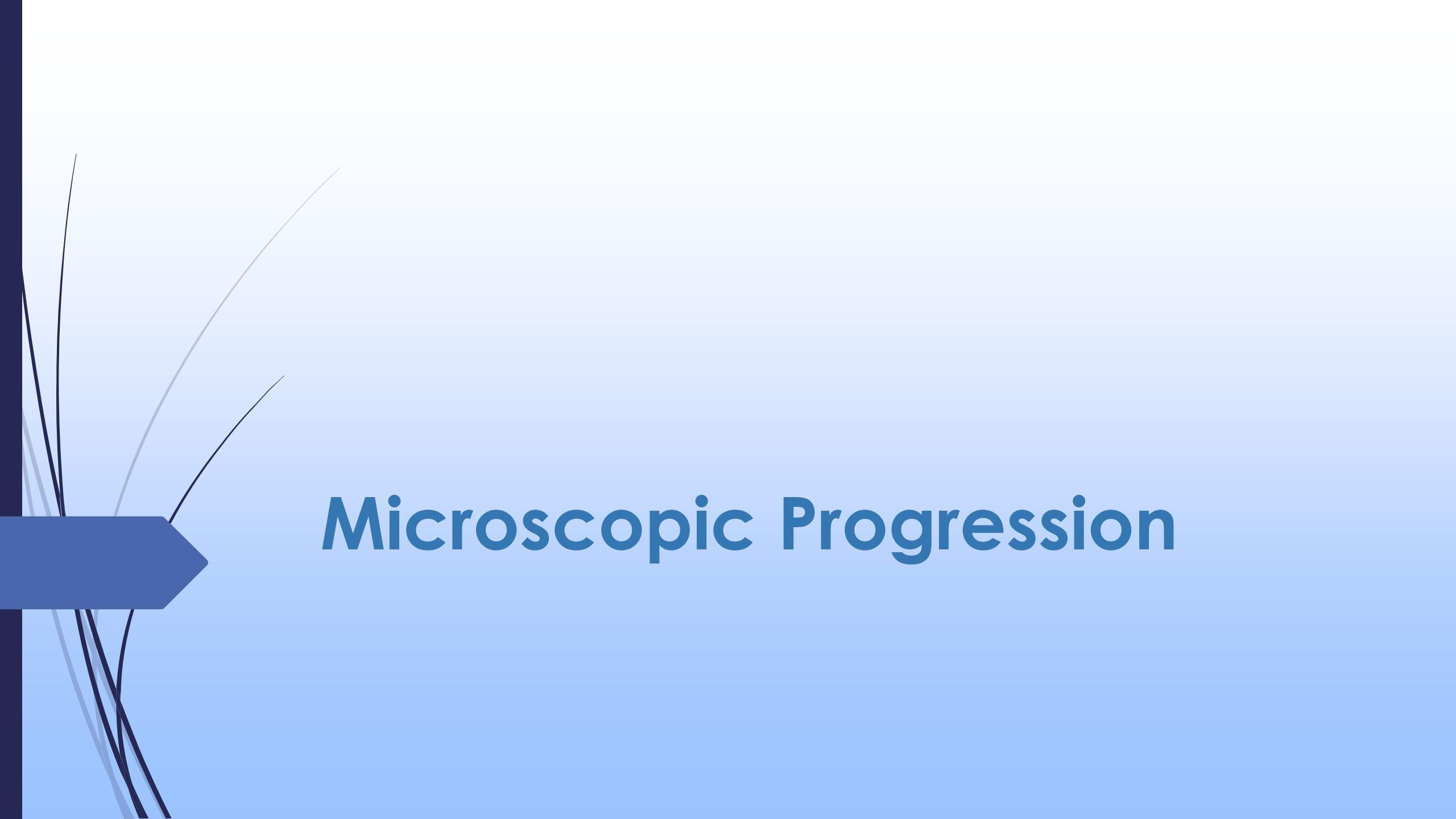
first I added the one's
and it ecwalled 12 so
I know theres another
ten and 2 one's So I'll
^{swap}
~~sure~~ the ten ones for
a ten so there's 6
tens and two ones
So it should ecwall
62.

First I added my
ones and it ecwalled
11 so I'll swap



When adding fractions, why don't I add the denominators?

$$1/8 + 3/8$$



Microscopic Progression



Microscopic progression

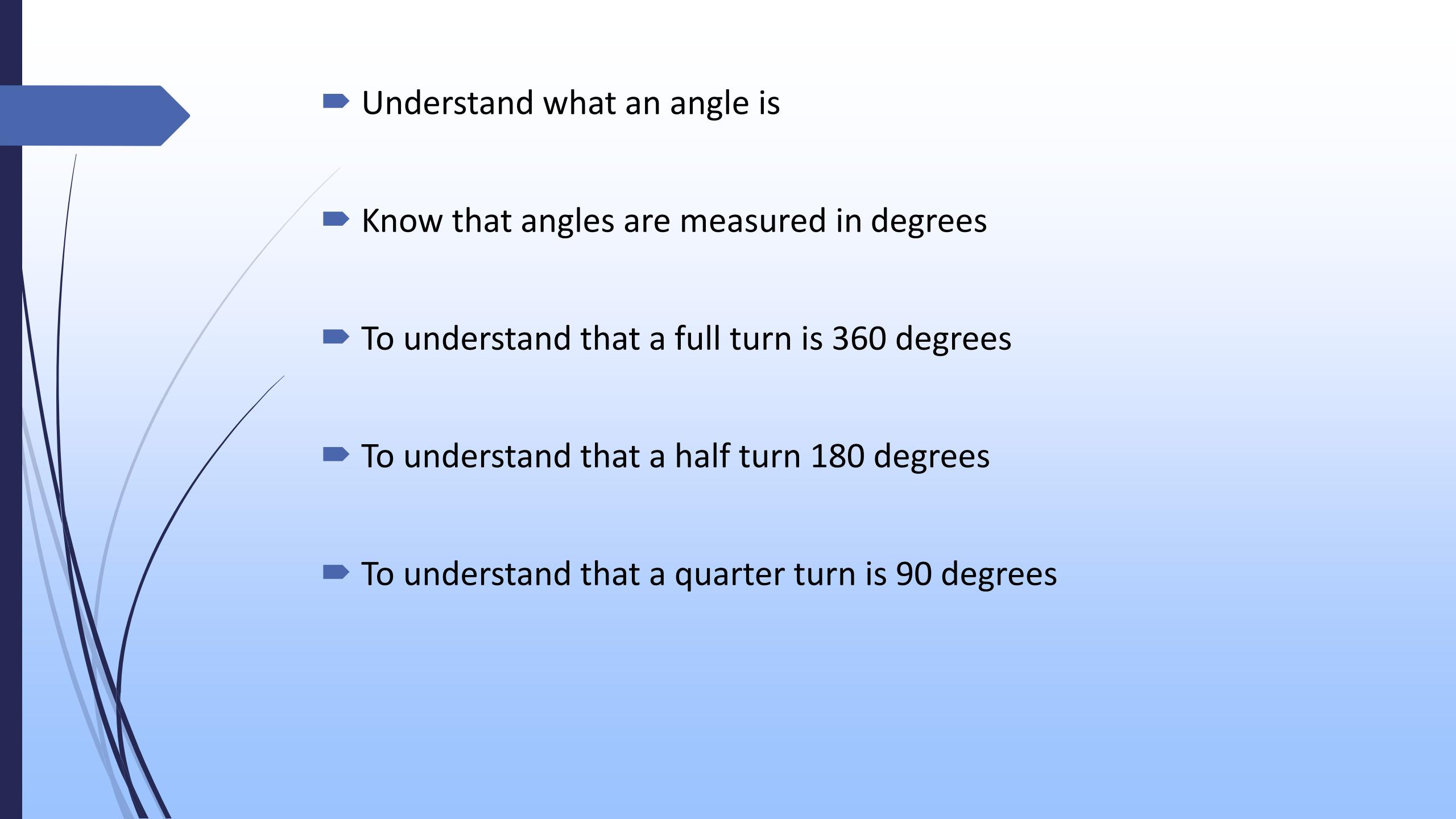
- ▶ The first element of teaching for mastery that we introduced to WPPS.
- ▶ What is the end point you want the children to reach?
- ▶ What are the tiny (microscopic) steps that a child would need to take to reach this?
- ▶ We aim to start at the very beginning and then identify any gaps in their understanding as we go.
- ▶ Organise immediate intervention for children who struggle with a step.
- ▶ Move through the steps at a speed appropriate for your children but not missing any step.
- ▶ All children move through the same set of steps regardless of speed or ability.

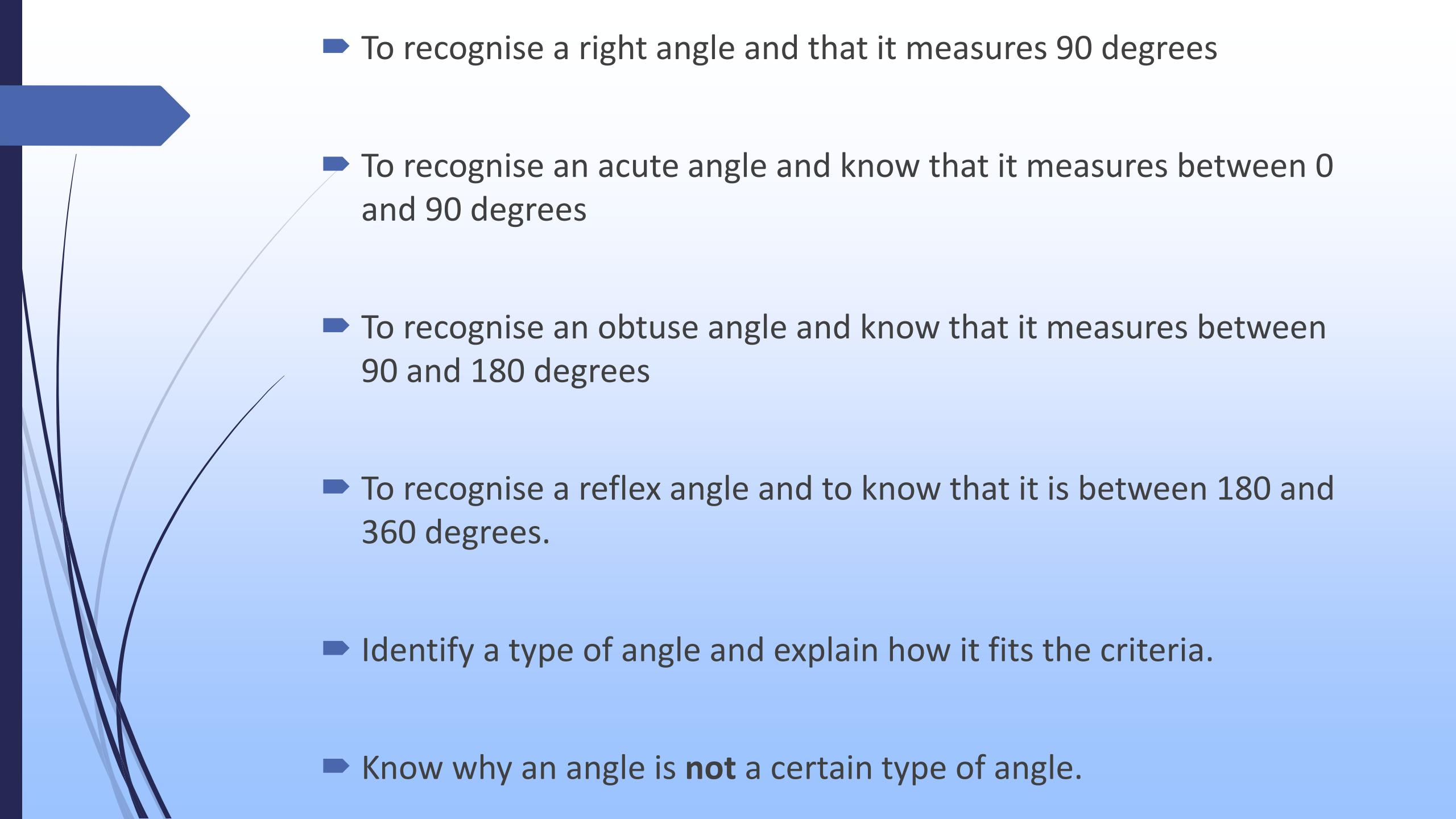


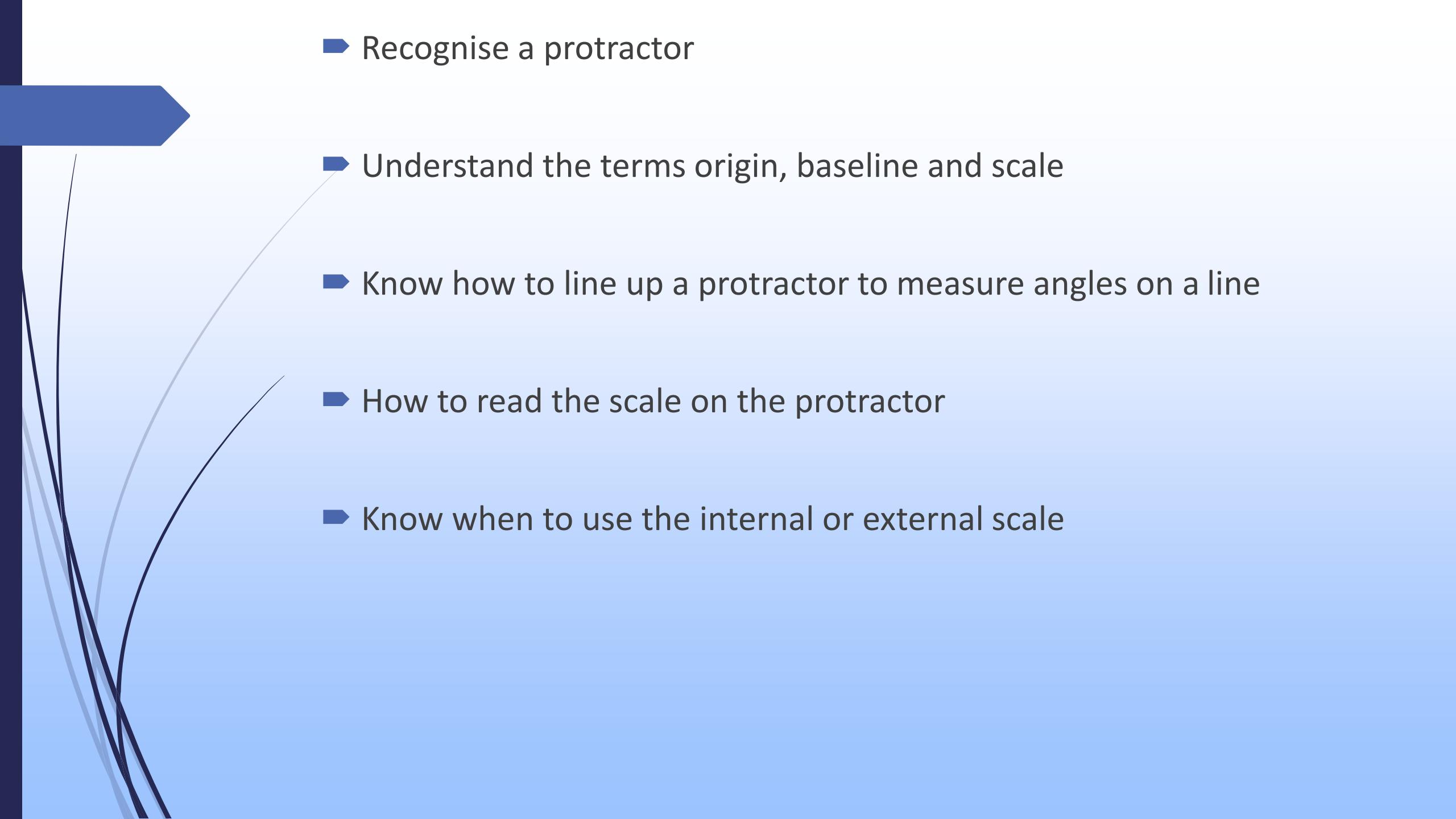
Target: To measure a given angle in degrees

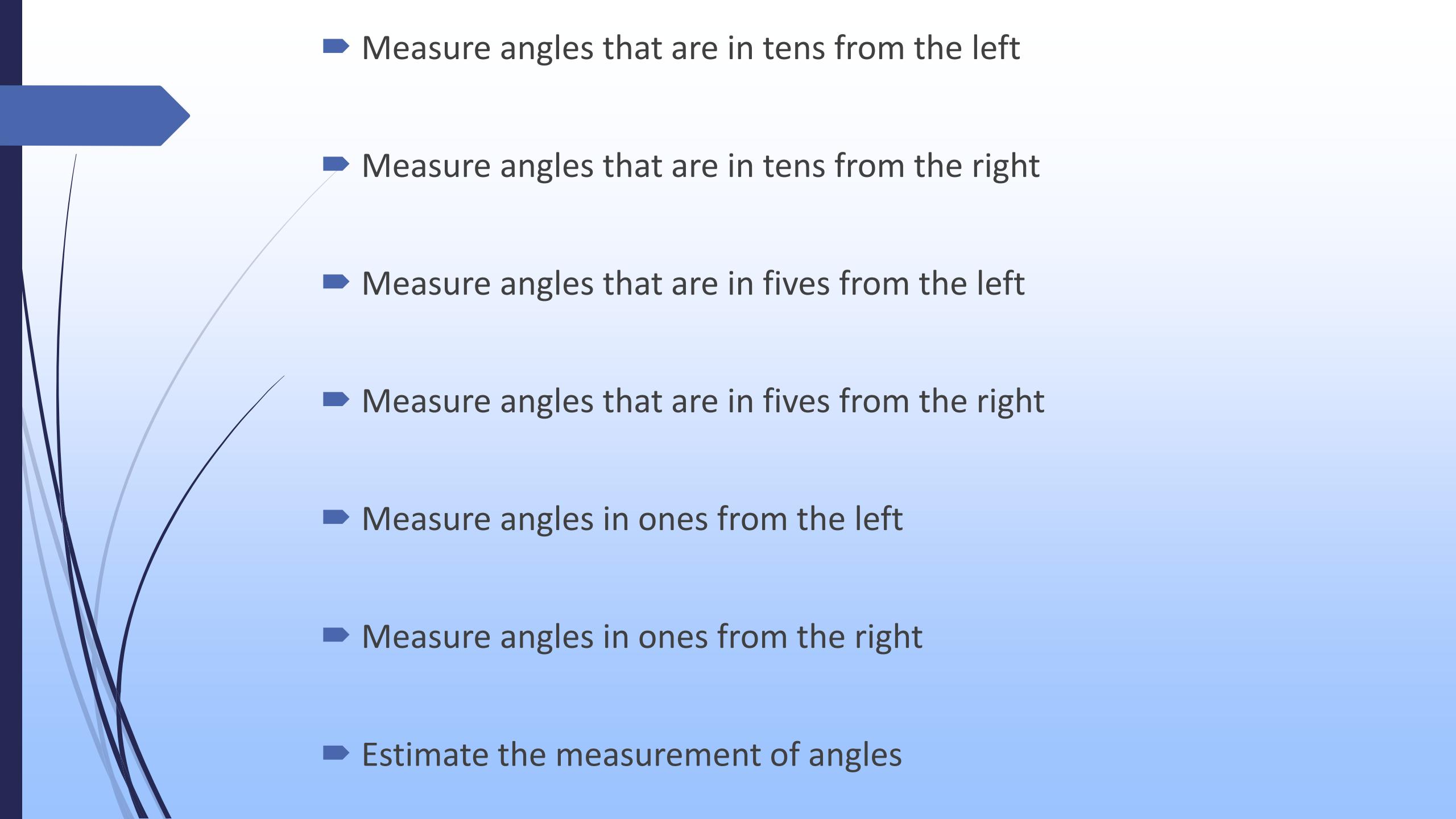
What is the end point you want the children to reach?

- ▶ What prior knowledge would a child need to be able to begin working on this target?
- ▶ What small steps would they need to take to fully achieve this target?
- ▶ How long do you think it would take a class to achieve this target fully?

- 
- ▶ Understand what an angle is
 - ▶ Know that angles are measured in degrees
 - ▶ To understand that a full turn is 360 degrees
 - ▶ To understand that a half turn 180 degrees
 - ▶ To understand that a quarter turn is 90 degrees

- 
- ▶ To recognise a right angle and that it measures 90 degrees
 - ▶ To recognise an acute angle and know that it measures between 0 and 90 degrees
 - ▶ To recognise an obtuse angle and know that it measures between 90 and 180 degrees
 - ▶ To recognise a reflex angle and to know that it is between 180 and 360 degrees.
 - ▶ Identify a type of angle and explain how it fits the criteria.
 - ▶ Know why an angle is **not** a certain type of angle.

- 
- ▶ Recognise a protractor
 - ▶ Understand the terms origin, baseline and scale
 - ▶ Know how to line up a protractor to measure angles on a line
 - ▶ How to read the scale on the protractor
 - ▶ Know when to use the internal or external scale

- 
- ▶ Measure angles that are in tens from the left
 - ▶ Measure angles that are in tens from the right
 - ▶ Measure angles that are in fives from the left
 - ▶ Measure angles that are in fives from the right
 - ▶ Measure angles in ones from the left
 - ▶ Measure angles in ones from the right
 - ▶ Estimate the measurement of angles

Microscopic progression for written methods.

▶ What small steps would you need to take to be able to solve this problem?

$$456 + 339$$

▶ What possible misconceptions might a child have?

 $456 + 339$

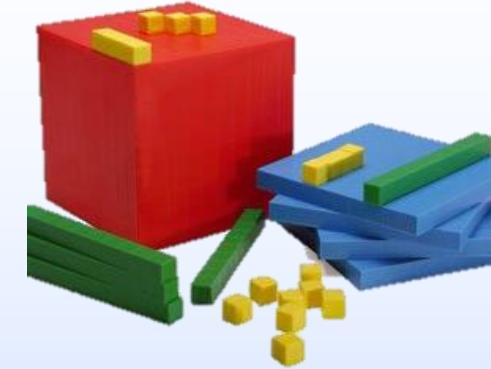
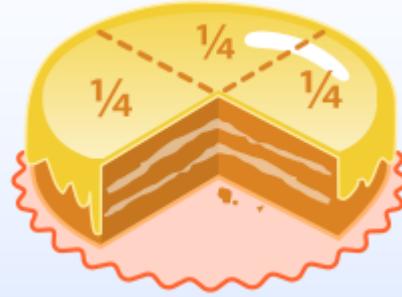
- They will already need to have mastered addition with no regrouping. They need to be confident with number bonds.
- The specific skill they will need to learn for this one is to carry a digit from the tens column to the ones column.
- They will need to understand that 15 is 5 ones and 1 ten.
- There are many possible misconceptions:
 - they could line up the numbers incorrectly
 - they could carry the 5 instead of the ten
 - they could add the numbers incorrectly.



Conjecture

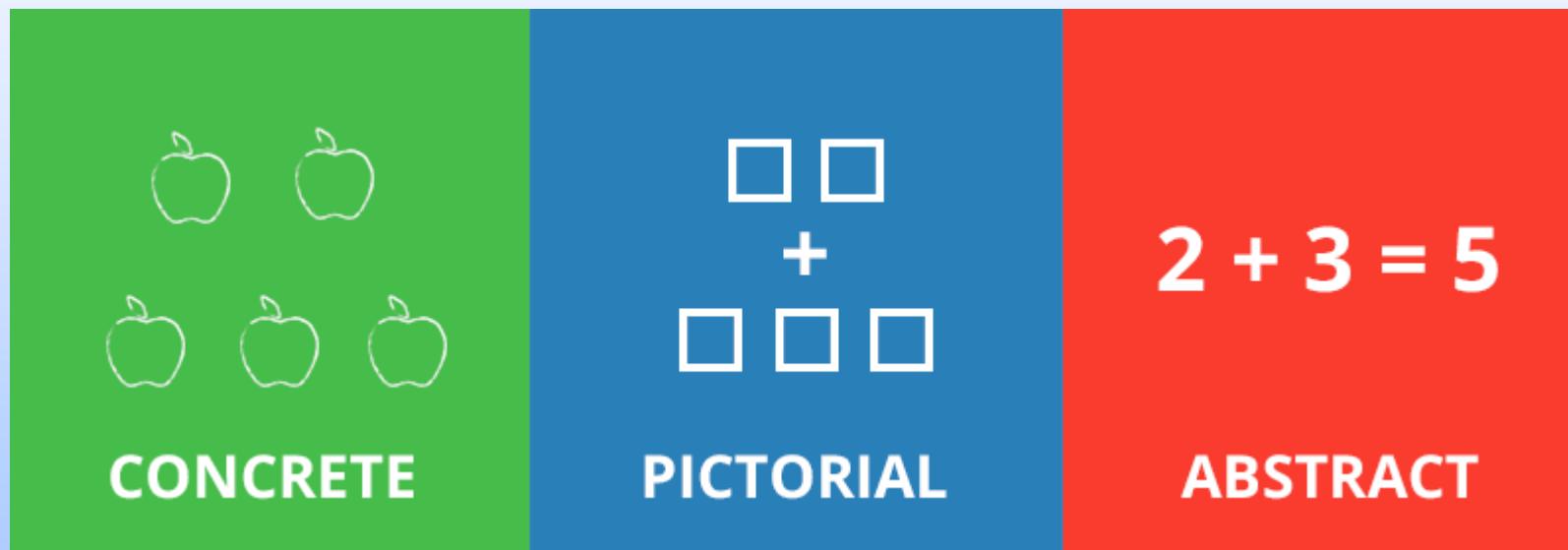
'To find a tenth of a number I divide by 10 and to find a fifth of a number I divide by 5.'

- ▶ Do you agree?
- ▶ Explain your reasoning.
- ▶ Can you apply this reasoning to any fraction?



Concrete and Pictorial Resources

What is the C-P-A approach?





C-P-A

- ▶ Pupils are unable to tackle abstract maths problems without being able to visualize the problem first.
- ▶ It is important for pupils to have a concrete foundation use to in a difficult problem.

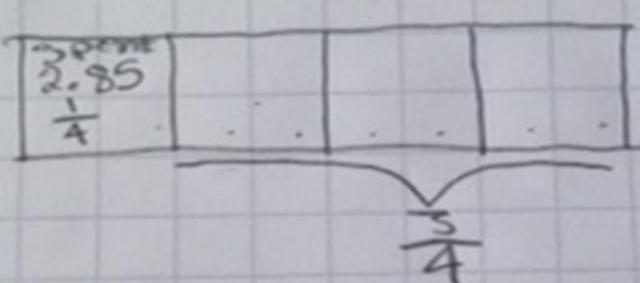
Pictorial representations and drawing.

- ▶ Children can use pictures to represent a problem. This often helps them see the answer.
- ▶ Drawing a picture is a really helpful resource to fall back on when you are not sure how to approach a problem.
- ▶ It is important to get the children comfortable with drawing bar models and other representations.

Problem Two

Lara had some money. She spent £1.25 on a drink. She spent £1.60 on a sandwich. She has three-quarters of her money left. How much money did Lara have to start with?

$$\begin{array}{r} \text{£ } 1.25 \\ \text{£ } 1.60 \\ \hline 2.85 \end{array}$$



$$\frac{1}{4} \text{ of her money} = \text{£ } 2.85 \checkmark$$

$$\begin{array}{r} 2.85 \\ \times 4 \\ \hline 11.40 \end{array}$$

We * times by 4 not 3 as you need to find 4 quarters.

Explaining understanding
pictorially

Reasoning and problem solving

WA5 - Recognise the relationship
between addition and subtraction

Sam worked out

$$82 - 45 = 47$$

Can you use the inverse to
check Sam's equation?

Is he correct? If not, where did
he go wrong?

$$45 + 47 = 92$$

$$40 + 40 = 80$$



$$5 + 7 = 12$$



he is incorrect
because $40 + 40 = 80$
then you exchange
the 4 tens for
1 ten then you have
and I have 2
ones left which is 92.

Explaining understanding
pictorially



A Typical Lesson

What you will see in Wimbledon Park.



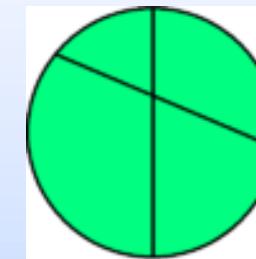
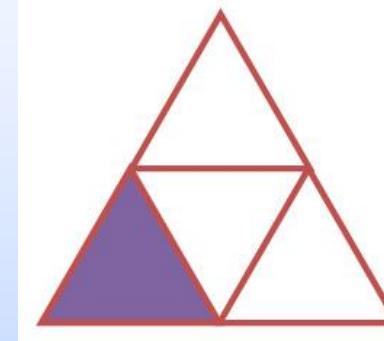
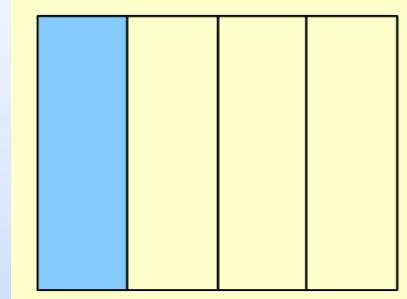
Variation

- ▶ When we present a collection of examples or exercises related to some mathematical idea to our pupils what do we want them to be looking for?
- ▶ What do we want them to pay attention to?

Conceptual Variation

- Varying the representation to extract the essence of the concept.
 - Supporting the generalisation of a concept, to recognise it in any context
 - Drawing out the structure of a concept – what it is and what it isn't
- ➡ *To find out what something is, we need to look at it from different angles – then we will know what it really looks like!*

Conceptual Variation



► You will often hear teachers ask the question:
“What’s the same, what’s different?”

Procedural Variation

- ▶ Procedural variation occurs within the process of doing mathematics
- ▶ Characteristics are small steps are made with slight variation
- ▶ This can be in the context of :
 - A mathematics lesson
 - An activity/practice exercise
 - Within the process of solving a problem

I can subtract a multiple of ten to any 2 digit number

Small steps that draw attention to one idea

Fluency

$$46 - 40 = 6 \quad \checkmark$$

$$46 - 30 = 16$$

$$46 - 20 = 26$$

$$46 - 20 = \underline{\quad}$$

$$46 - 10 = 36$$

$$46 + 10 = 56 \quad \checkmark$$

$$46 + 20 = 66 \quad \checkmark$$

$16 + 30 = 76$

$$16 + 40 = 86$$

Reasoning

What do you notice about the tens and ones in 46 and the tens and ones in your answer? Why

What is this?
When I add or subtract
tens digits changes
when I added or subtracted
Subtracted tens but the
one's digit is always
the same because I didn't
add or subtract any ones!

Questioning and Ping Pong Teaching

- ▶ Lessons are well paced with clear progression of learning throughout the lesson.
- ▶ Activities move rapidly from teacher to children and back throughout the lesson.
- ▶ Teacher leads back and forth interaction, including questioning, short tasks, explanation, demonstration, and discussion.
- ▶ Teacher uses carefully thought out questions including “What’s the same, what’s different?”, “How do you know?” and “Can you prove it?” to lead pupil discussion.
- ▶ Children have time to explore some concepts themselves or with their partners in the form of short “talk tasks”.

Reasoning and talk for maths

- ▶ All children will be taught and expected to use correct mathematical vocabulary consistently across the school.
- ▶ Children are expected to back up their answers by reasoning and proof.
- ▶ Correct vocabulary is modelled and prompted by the teacher.
- ▶ Children will be encouraged to explain their methods – if they cannot explain how to do it they have not mastered it.
- ▶ Books provide evidence of reasoning.

Reasoning

What generalisation can you make
about odd and even numbers?

Even numbers ~~only always~~ have 2, 4, 6,
8 or 0 at the end ~~or~~ which is
the ones column. Even numbers
can always be shared equally
between two.

Odd numbers always have 1, 3, 5, 7, 9
at the end which is the ones
column. Odd numbers can never be
shared equally between two.

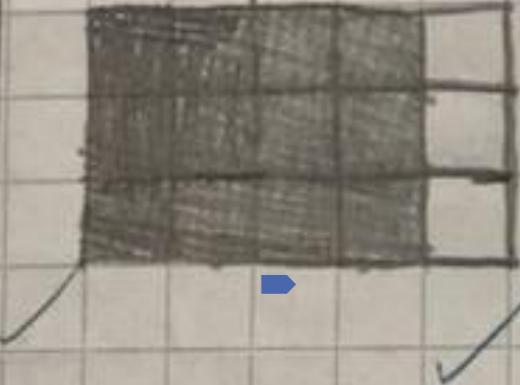
Reasoning

True or False?

"One ten and twelve ones is bigger than two tens."

Explain how you know!

True because if you have a ten and a
hother ten ~~+~~ that would make 20 so
if you had one ten and twelve ones
if you ~~has~~ just count the tens you
will have 2 tens and then you will have 12 ones
left so that would make 22.



A fraction is an amount or a part of a number or a thing. This particular fraction is $\frac{12}{15}$. A denominator is what the fraction is out of. $\frac{12}{15}$ denominator A numerator is how much you have, in this case, it is 12. I have 12 of the fraction because it is $\frac{12}{15}$ and the numerator is how much you have; I have 12. You can draw a diagram in a square, rectangle or circle. If you want to turn it into a decimal you have to divide the numerator by the denominator, in this case: $15 \overline{)12.0}$ so $\frac{12}{15}$ is equal to 0.8 because 12 divided by 15 is 0.8. Different types of fractions are:

- Mixed numbers
- Improper fractions
- Simple fractions.



True or False?

$$20 \times 100 = 2 \times 10 \times 10$$

Tell your partner why!



Supporting all learners



Supporting all learners

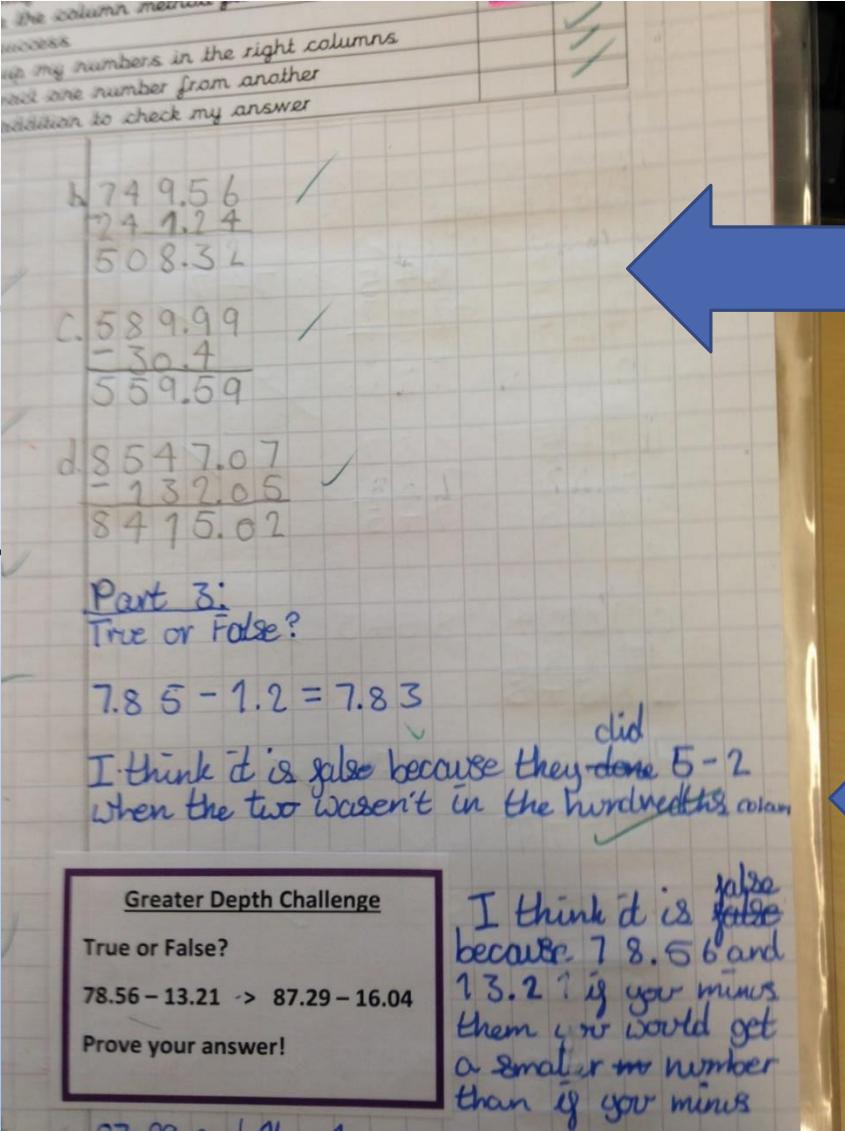
- ▶ Same input for all children
- ▶ Ping pong style teaching – all children get a chance to respond
- ▶ Group work and talk tasks
- ▶ Scaffolds – word bank, extra concrete resources, TA in small group
- ▶ Carefully planned questions to address misconceptions
- ▶ Carefully planned questions to challenge quick graspers



Supporting all learners

- ▶ Children work on the same independent questions which will be a mix of fluency, problem solving and reasoning.
- ▶ The tasks typically get more challenging as the children move through them.
- ▶ Children who are struggling may be in a small group to work on one or two tasks with a TA or teacher.
- ▶ Children who are comfortable with the learning will be able to choose a “Greater Depth” challenge which will challenge them further while still focusing on the same learning objective.

Progression of tasks.



Fluency

Reasoning: Talk for Maths

Greater Depth Challenge ✓

Always true, sometimes true or never true?

If I have a two digit number and I add ones to it, I will only need to change the ones digit.

Examples $21 + 5$

$28 + 5$

Extending
confident learners

This is sometimes true because you don't have to bridge through 10 when you add $21+5$ or $22+5$ but other times you have to bridge through 10 like $28+5$ or $29+5$. The change if you bridge through ten.

This is a good explanation.

Year 5 and 6

- ▶ Although the different maths classes might move at different speeds they all follow the same microscopic steps and all children access the same learning.
- ▶ The same independent tasks are given to all three classes.



Home Learning

- ▶ Pupils are securing concepts and procedures that have been covered in class.
- ▶ The aim is not to set ‘new’ work.
- ▶ Work on securing knowledge, fluency and automaticity of skill



Thank you for your support!